Modeling interactions between the quasi-geostrophic vertical motion and convection in a single column

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Part I: a single-column modeling framework: interaction between LS and convection

Part II: applications on the 2010 Pakistan extreme precipitation events
Introduction:
the idea of modeling tropical precipitation in a single column
(Sobel and Bretherton 2000, Raymond and Zeng 2005, Kuang 2008,
Romps 2012, …)

\[ \partial_t T = Adv_T + \frac{\sigma p}{R} w + Q, \]  
\[ \partial_t q = Adv_q - s_q w + Q_q. \]  

convective heating

convective moistening
Introduction:
The idea of modeling tropical precipitation in a single column (Sobel and Bretherton 2000, Raymond and Zeng 2005, Kuang 2008, Romps 2012, …)

\[
\partial_t T = Adv_T + \left( \frac{\sigma p}{R} w + Q \right), \quad \text{convective heating}
\]
\[
\partial_t q = Adv_q - s_q w + Q_q, \quad \text{convective moistening}
\]

Q and w have to be solved simultaneously.

The weak temperature gradient (WTG, Sobel and Bretherton 2000):

\[
W_{wtg} \frac{\partial \bar{\theta}}{\partial z} = \frac{\bar{\theta} - \theta_{ref}}{\tau},
\]
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(WTG, Sobel and Bretherton 2000):

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Applications: convection responses to ENSO (e.g. Chiang and Sobel 2002); Seasonality (Gentine et al. 2015); QBO (Nie and Sobel 2015), …
However, what about the extratropics?
From now on, we use the equations in $p$ coordinate (see Appendix A) as following:

$$\partial_p p w + f_0 r^2 w = f_0 \partial_p (\text{Adv} \downarrow F)$$

where

$$\text{Adv} = \bar{T}_p \partial_p \ln \bar{\sigma}.$$ 

We can denote the advective forcing terms:

$$F_\downarrow = f_0 \partial_p (\text{Adv} \downarrow F),$$

$$F_T = R_p f_0^2 (\text{Adv} T).$$

The temperature and moisture equations are

$$\partial_t T = \text{Adv} T + \frac{\sigma p}{R} w + Q,$$

$$\partial_t q = \text{Adv} q - s_q w + Q_q.$$ 

Some useful derivations. Plug thermal dynamic equation back to replace $Q$, we have

$$\partial_p w = F_\downarrow R_p f_0^2 \partial_t T.$$ 

Plug thermal dynamic equation back to replace $w$, we have

$$\partial_p \left( \frac{\sigma p}{R} w \right) = \partial_p Q + f_0 \partial_p \text{Adv} T.$$ 

For now we only consider the boundary condition as $w = 0$ at bottom and top. Given a horizontal wavenumber $k$, the above equation becomes

$$\partial_p w = F_\downarrow + R_p \left( k f_0 \right)^2 \partial_t T.$$ 

Convective heating

$$\partial_t T = \text{Adv} T + \frac{\sigma p}{R} w + Q,$$

Convective moistening

$$\partial_t q = \text{Adv} q - s_q w + Q_q.$$ 

A closure (super-domain scale parameterization) that relate the large-scale vertical motion with the states of the local column:

Tropics:

$$\text{WTG}$$

Extratropics:

$$?$$
From now on, we use the equations in $p$ coordinate (see Appendix A) as following:

$$\frac{\partial}{\partial p} w + f_0^2 R_\text{pf} \left( \text{Adv} \right)_R \frac{\partial}{\partial p} \left( \text{Adv} \right)_R = \text{Adv} \left( \text{Adv} \right)_R \frac{\partial}{\partial p} \left( \text{Adv} \right)_R.$$  \hspace{1cm} (36)

We can denote the advective forcing terms:

$$F_\text{Adv} = \frac{\partial}{\partial p} \left( \text{Adv} \right)_R,$$  \hspace{1cm} (37)

$$F_T = R_\text{pf} \frac{\partial}{\partial p} \left( \text{Adv} \right)_R.$$  \hspace{1cm} (38)

The temperature and moisture equations are

$$\frac{\partial}{\partial t} T = \text{Adv} T + \frac{\sigma p}{R} w + Q,$$  \hspace{1cm} (39)

$$\frac{\partial}{\partial t} q = \text{Adv} q - s_q w + Q_q.$$  \hspace{1cm} (40)

Some useful derivations. Plug thermal dynamic equation back to replace $Q$, we have

$$\frac{\partial}{\partial p} w = F_\text{Adv} + \frac{\partial}{\partial t} T.$$  \hspace{1cm} (41)

Plug thermal dynamic equation back to replace $w$, we have

$$\left( \frac{\partial}{\partial p} + \frac{f_0^2}{R_\text{pf}} \right) \frac{\partial}{\partial t} T = \frac{\partial}{\partial p} Q + \frac{\partial}{\partial t} \text{Adv} T.$$  \hspace{1cm} (42)

For now we only consider the boundary condition as $w = 0$ at bottom and top.

Given a horizontal wavenumber $k$, the above equation becomes

$$\frac{\partial}{\partial p} w = F_\text{Adv} + \frac{\partial}{\partial t} T.$$  \hspace{1cm} (43)

$$\frac{\partial}{\partial p} w = F_\text{Adv} + \frac{R_\text{pf} \frac{f_0^2}{R_\text{pf}}}{k} \frac{\partial}{\partial t} T.$$  \hspace{1cm} (44)

$$\left( \frac{\partial}{\partial p} + \frac{f_0^2}{R_\text{pf}} \right) \frac{\partial}{\partial t} T = \frac{\partial}{\partial p} Q + \frac{\partial}{\partial t} \text{Adv} T.$$  \hspace{1cm} (45)

where

$$F_\text{Adv} = \frac{\partial}{\partial p} \left( \text{Adv} \right)_R,$$  \hspace{1cm} (46)

$$F_T = R_\text{pf} \frac{\partial}{\partial p} \left( \text{Adv} \right)_R.$$  \hspace{1cm} (47)

For convective heating

$$\partial_t T = \text{Adv} T + \frac{\sigma p}{R} w + Q,$$  \hspace{1cm} (39)

For convective moistening

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a closure (super-domain scale parameterization) that relate the large-scale vertical motion with the states of the local column:

tropics:

WTG

extratropics:

QG-omega
quasi-geostrophic omega equation:

\[ \partial_{pp} w + \frac{\sigma}{f_0^2} \nabla^2 w = -\frac{1}{f_0} \partial_p (Adv\zeta) - \frac{R}{pf_0^2} \nabla^2 AdvT - \frac{R}{pf_0^2} \nabla^2 Q, \]

\[ \omega_{total} = \omega_\zeta + \omega_T + \omega_Q \]

\[ \omega_{QG} \]
\[ \partial_{pp}w + \frac{\sigma}{f_0^2} \nabla^2 w = -\frac{1}{f_0} \partial_p(\text{Adv}_\zeta) - \frac{R}{pf_0^2} \nabla^2 \text{Adv}_T - \frac{R}{pf_0^2} \nabla^2 Q, \]

**longwave limit:** middle-latitude dry dynamics (dry QG)

\[ \omega_{\text{total}} = \omega_\zeta + \omega_T + \omega_Q \]

**shortwave limit or f->0:** tropical dynamics (Strict WTG)

\[ \omega_{\text{total}} = \omega_\zeta + \omega_T + \omega_Q \]
\[ \partial_{pp}w + \frac{\sigma}{f_0^2} \nabla^2 w = -\frac{1}{f_0} \partial_p(\text{Adv}_\zeta) - \frac{R}{p f_0^2} \nabla^2 \text{Adv}_T - \frac{R}{p f_0^2} \nabla^2 Q, \]

\[ \omega_\zeta + \omega_T \sim \omega_Q \]

**wavelength:** roughly between 700km and 2000km

**extratropics:** plenty of QG disturbances

**strong precip.:** the convective heating is significant
the modeling framework: coupling the large-scale dynamics and convection with the QG-omega equation:

$$
\partial_t T = Adv_T + \frac{\sigma p}{R} w + Q,
$$

$$
\partial_t q = Adv_q - s_q w + Q_q.
$$
a. assume there is a characteristic length scale:

$$
\partial_{pp} w - \sigma \left( \frac{k}{f_0} \right)^2 w = -\frac{1}{f_0} \partial_p (Adv_\zeta) + \frac{R}{p} \left( \frac{k}{f_0} \right)^2 Adv_T + \frac{R}{p} \left( \frac{k}{f_0} \right)^2 Q.
$$
the modeling framework: coupling the large-scale dynamics and convection with the QG-omega equation:

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b. use a single column model or a cloud resolving model to simulation convection

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c. prescribe QG forcing (Adv_\zeta, Adv_T, Adv_q,...)
the modeling framework:

large-scale perturbations

\[ \text{Adv}_\xi, \text{Adv}_T, \text{Adv}_q, \omega_{\text{topo}} (\text{topographic forcing}) \]

... 

forcing 

large-scale omega

feedback \( \omega_Q \)

Convection
Part I: a single-column modeling framework: interaction between LS and convection

Part II: applications on the 2010 Pakistan extreme precipitation events
Northern Pakistan floods during monsoon seasons:

2003
2007
2010
2011
2012
2013
2014

Nie et al. 2010
2010 Pakistan flood events: ERA-interim Precip.
Q: What causes the extreme precipitation?
Obs. $\omega_Q > \omega_\zeta + \omega_T$

3D QG-omega inversion:
Obs.

topographic wind (lower b.c.):

\[ \omega_{PBL} \approx V_{g,PBL} \cdot \nabla h_0 = \omega_{\text{topo}} \]
\[ \partial_{pp} w + \frac{\sigma}{f_0^2} \nabla^2 w = -\frac{1}{f_0} \partial_p (Adv \zeta) - \frac{R}{pf_0^2} \nabla^2 Adv_T - \frac{R}{pf_0^2} \nabla^2 Q, \]

3D inversion

\[ \nabla^2 \sim -k^2 \]

k \approx \frac{2\pi}{1000 \text{km}}

1D inversion
Model: \(\text{Adv}_\zeta+\text{Adv}_T+\text{Adv}_q+\omega_0\)
Obs.

Model: $\text{Adv}_\zeta + \text{Adv}_T + \text{Adv}_q + \omega_0$

$w$ (hPa/hr)

$\omega$ (hPa/hr)
Summery:

- convection + QG-omega in single column modeling

- Using this modeling framework, we reproduces the 2010 Pakistan flood events quite well.
  * the coupling between convection and large-scale dynamics is important.
  * the topographic wind accounts for the triggering the extreme events in these event.

Thank you.